Marketing Letters 10:3 (1999): 233–248

Bounded Rationality Modeling

BERTRAND MUNIER
GRID, Ecole Normale Supérieure de Cachan

REINHARD SELTEN
Emeritus Professor, University of Bonn

D. BOUYSSOU
ESSEC

P. BOURGINE
CREA, Ecole Polytechnique

R. DAY
University of Southern California

N. HARVEY
University College London

D. HILTON
Université de Toulouse-II

M. J. MACHINA
University of California at San Diego

PH. PARKER
INSEAD

J. STERMAN
M.I.T.

E. WEBER
Ohio State University

B. WERNERFELT
M.I.T.

R. WENSLEY
University of Warwick

Abstract

This paper deals with bounded rationality as a way to describe behavior and focuses on the question of how to build such boundedly rational models. The first part is a discussion of the reasons why such models are needed and on the situations in which they can be regarded as more particularly useful. The second part examines three strategies of research towards bounded rationality modeling which have emerged in the last ten years and weights them. The concluding remarks offer a first link between the respective typologies of strategies and of situations and calls for additional experimental work by marketing scientists and economists together.

Key words: Decision-making, consumer behavior, procedural rationality, choice functionals, adaptive behavior

Introduction

Many models of consumer behavior make explicit or implicit use of a 'strong' rationality assumption whereby consumers maximize some type of a uni- or multi-attribute utility function. Herbert Simon has cast doubts about this representation of the rational behavior of economic agents. First, in the Fifties, Simon suggested that agents would consider some threshold of satisfaction rather than maximize a utility function (Simon, 1955). Most management scientists and economists still refer today to that seminal intuition. However, in a famous paper (Simon, 1978), Simon gave a different interpretation of bounded rationality: rather than behaving as utility maximizers, economic agents follow some reasonable procedure, or sequence of thoughtful steps when they decide on whatever issue. The behavior of economic agents is governed by a 'procedural rationality' rather than being the product of some 'substantive rationality'. The essence of bounded rationality is thus to be a 'process of thought' rather than a 'product of thought': Individuals have recourse to reasonable procedures rather than to sophisticated computations which are beyond their cognitive capacities. In the mid-Nineties, Simon added that these procedures, however varied they may be, are to be characterized by at least two stages: *recognition* and *heuristic search* through spaces of possibilities. This is all it takes, according to him, 'to explain not only everyday problem solving but also such phenomena as intuition, insight, and the cognitive aspects of creativity' (Simon, 1995).

These are the three levels of the increasingly deeper apprehension of the concept of bounded rationality which Simon provided us with and contrasted with the economic modeling of rational actors (Simon, 1986). Yet, one is stunned by the contrast between a fairly wide approval of these views, well-documented evidence provided (mostly by psychologists), on one hand, and on another hand the limited number of rigorous formal economic or marketing models devoted to operationalize them. This fact might be due to insufficient modeling of the ways we *represent* problems in our minds, of the way we *learn* to solve these problems, of the way *cognition* is to be confronted with affect and perhaps even with physiological questions amidst these problem solving tasks. It is precisely the purpose of this paper to focus on some of these issues and suggest into which channels research efforts should be fueled to let models of marketing, of economics, and of other social sciences make progress. The paper examines why this research program makes sense and what type of situations can be more particularly investigated (section 1). It then looks at strategies which are being used to implement such a research program (section 2) while section 3 concludes.

1. Why and How Bounded Rationality Models: Discussion and Typology

1.1. Basic discussion

Many economists do not agree on the significance of the bounded rationality concept and above all on the usefulness of modeling it. To refer to two contemporary illustrious economists, let us quote Maurice Allais and David Kreps:

> '*La psychologie des hommes est invariante dans le temps et l'espace, autrement dit elle peut être représentée par des fonctions invariantes'*
> (Human psychology is invariant in time and space, in other words it can be represented by invariant functions, in: Allais (1974, p. 114)).

and:

> '*The assertion that economists ought to come to grips with boundedly rational behavior, better to model important economic phenomena, is thus far from proven. There is no logical proof that we need to do this — no empirical phenomenon can be quoted that cannot somehow be rationalized within the realm of hyper-rationality — and substantial costs will be incurred if we try*' in : Kreps (1997, p. 171).

Three observations need be brought up regarding these two quotations, which help to delineate farther the topics we have to deal with here:

a) As the two quotations point out, arguments about bounded rationality arise only when it comes to *descriptively* model some agent's behavior — whether consumer's or business manager's. *Normative* models — telling us what decision criterion we should use if we want to pay tribute to a given set of axioms — are immune by nature to bounded rationality objections. On the other hand when the issue is of a *prescriptive* nature, that is, for example, is about to make a decision in a business within a given environment, there will be little dispute that there will not be, in general, an *invariant* function which, by maximization, would be sufficient to yield a solution to all possible decision problems in the corportion. Hence, *descriptive* models (for example of consumers' behavior which, incidentally, is one major issue of interest in marketing problems) are the only class of models for which there can be and there is actually a controversy on the bounded rationality issue. In this respect, causal introspection, experimental evidence, many of the 'anomalies' (as they were called by Thaler in *JEP* for years) and the like, all speak in favor of something different from the strongly rational behavior as defined by Allais or Kreps.

b) In both quotations, the 'as if ...' argument of Milton Friedman implicitly emerges. What is argued here is that the question is not to describe *why* people decide, not to carefully follow the procedure by which they come to their conclusions, but to *represent what* people decide through the trick of maximizing some function. Of course, there are more 'radical' views on strong rationality, holding that there is no behavior which would not be pushing our innate utility to its maximum under the constraints imposed on the decision maker, but we do not deal with them here.

   In the less radical view retained here, it is important to realize that the maximization operator is not necessarily the most discriminating term between strong and bounded rationality modeling. All *depends on what is to be maximized* in the descriptive model : taking proxies of a rudimentary nature as arguments of the utility function will sometimes explain why consumers make decisions which look to them to be 'the best ones' whereas they are not. Examples can be found in Wernerfelt (1995) or in Prelec, Wernerfelt and Zettelmeyer (1997). One has to be careful, however, and refrain from building *ad hoc* models.

Indeed, the *invariance* of the function is a key criterion there: if the description of behavior requires changing, each time the problem changes, either the value of coefficients (context effects, for example) or — worse — the specification or the definitional space (strong inference effects, change of selected proxies, for example) of the function to be maximized, the 'strong' rational scheme of behavior becomes then irrefutable and looses scientific strength. Incidentally, isn't this what Kreps has in mind when he talks about 'somehow rationalizing' every empirical phenomenon by keeping *some* function but changing it according to the phenomenon? To him, this sounds to be 'hyper rationality', to us, *ad hoc* strong rationality. It seems to us that modeling the (boundedly rational, in this view) underlying procedure or the behavioral scheme should help avoiding mistakes. It also seems that the costs incurred in doing so should not necessarily exceed the costs of finding how to predict for each new problem the required changes in the specification of the function to be maximized. In this sense, a theory of bounded rationality should start with assumptions about the underlying procedure.

Abdellaoui and Munier (1994, 1998a, 1998b) have experimentally tested the problem-dependence of preference functionals in several experiments. Their results show that preferences towards risk clearly depend on the 'risk-structure' faced, that is, change according to the type of probability distribution faced. In some sense, these results provide experimental evidence to (a generalization of) lottery-dependent utility (Becker and Sarin, 1987). Thus, rational behavior seems to happen in a Simonian two stage procedure: cognition, heuristic search (Munier, 1999). The *specification* of the representation functional might however not change substantially and thus the attitude towards risk which emerges from these experiments can be to a large extent described by a set of rank dependent models endowed with different sets of coefficients values. To each risk structure, its coefficients values. Admittedly, these results bear on simple choices between three outcome risky lotteries. They do not deal either with the question of the underlying procedure.

c) Among Kreps' arguments, the least impressive is certainly that 'there is no *logical* argument . . .'. First, there is no logical argument *against* bounded rationality either. Second, it seems that, contrary to *normative* decision models, *descriptive* ones will hardly admit any logical argument. Opponents to bounded rationality implicitly admit the convergence of the infinite regress of folding search costs into a problem: But could there be a *logical* argument to the effect that sequences of the *whole* class of such models *have to* converge? Is not the only class of decision models for which logical arguments make sense the one of normative models?

Other arguments dealing with the reasons why one should model bounded rationality can be found in the survey by Conlisk (1996, pp. 684–686), notably the one according to which markets will eliminate low rationality agents and thus low rationality, etc. These arguments have not been discussed in the session and will not be here.

1.2. A typology of the relevant situations involved.

It is usually acknowledged that one cannot judge an observed behavior as being 'rational' or not as long as the situation in which that behavior takes place has not been described, in particular in relation to its information structure and to the computational means available to economic agents. To discriminate between two different schemes of rationality, there is a minimum of agreement required on the description of the situation considered (Machina, 1989, p. 1662). Well-trained economists know, for example, that incomplete markets can put an extremely hard strain on agents looking for optimal decisions, in comparisons to what is required under a complete set of markets. In accounting for human cognitive capacities, four types of situations can be contrasted to give a more precise account of the realm of bounded rationality:

1) *Situations endowed with programmable decisions (in the sense of Simon, i.e., where algorithms can be used to find the most appropriate solution)*: In such situations, we might, as a rule, dispense with any kind of bounded rationality modeling. A typical example is inventory management in long lived firms with well-monitored markets. In practice, some models can however be questionable, when relying more on analytical convenience than on conformity to empirical evidence. Bounded rationality modeling might then help overcome such flaws.
2) *Situations where evolved task-specific procedures are appropriate*: these are typically situations of relatively limited complexity in the sense where interactions between specifically relevant variables do not entirely escape the reasoning of agents. Usually, agents find at least qualitatively appropriate moves for the task they have to perform. A non economic example could be golf playing. An economic example could be auction markets. In limitcases, however, agents fail to identify — even qualitatively — the appropriate moves (see the Bazerman-Samuelson task, below).
3) *Situations where evolved task-general procedures are helpful* (heuristics, chunks): In these situations, agents have difficulty finding even qualitatively appropriate responses to the challenges arising from the task because, even if they spot the interactions between relevant variables, the specifics of these interactions are beyond the grasp of their intuition. The agents are then left with heuristics like availability (similarity with a case easy to recall) or representativeness (similarity with some benchmark), which help the decision maker find its way around. An economic example would be making forecasts using time series of the past (Bolger and Harvey, 1998). Harvey (1999), using the framework of Armstrong's forecasting stages (1985), gives a number of clues as to improve the procedure used in forecasting tasks. A non economic example might be chess playing: Although the computation of the optimal solution is out of reach, chunks and heuristics on relatively limited series of moves help the individual to find how to win.

4) *Situations where evolved task-general procedures are of little help*: Typically, these are situations where the individual is lost and very seldomly find appropriate answers to the challenges met. Trying to control highly volatile dynamic systems with positive time-lagged feed-backs, multiple agents, etc., is an example. Managers facing endogenous fluctuations of the (generalized) famous cobweb-type together with time-lags (through some monetary constraint due to previous year results, for example) are in such situations. Laboratory experiments show that agents do not at all master these situations (Kampmann and Sterman, 1998). Not only are optima badly 'peaked', but heuristics or logical reasoning of the human mind generally sends the individual way below any reasonable score: they are of an insufficient variety to be able to deal, even approximately, with such tricky systems.

In situations of type 1, strong rationality models can be used to describe behavior, provided that modeling is adequate, as already pointed out. In contrast, in situations of type 2 and 3, we can hypothesize that although some set of preference functionals may be appropriate to *describe* moves taken by economic agents in some cases (for example: risky lottery preferences experiments, see above), it will generally not be the case. This will be hardly possible at all in situations of type 4, close to chaotic cases. In the latter situations, neither instinct nor any kind of heuristics will help individuals find their way out and modelers will have little choice but renouncing rational models in favor of random time series or something of the type, if they want to describe behavior.

Bounded rationality modeling may thus be at its best in situations of types 2 and 3 although a look at the decision procedures at work might certainly help to understand behavior in even the most simple situations (type 1).

Bounded rationality could therefore be defined as *the design of reasoning procedures resorted to by the human mind when making decisions between two worlds: the simple one and the chaotic or near-chaotic one.* Trying to 'reconstruct' behavior in these situations through a functional taking its maximum at the solution point will generally imply to pick up a different functional for each new problem. *If a stable* functional were to do the job in each case, then Kreps's argument could be convincing (see the discussion in 1b above).

It is noteworthy that, in this view, the procedure leads to the 'real' choice, not the maximization of a revealed preference functional which is, at best, a tool to reconstruct the choice, in general a handy way to approximate the real solution. This does not mean, of course, that using such functionals cannot be helpful in many cases.

Which variables, then, will presumably have to be considered in bounded rationality models, with what kind of relationships between themselves and what impacts on decision making? It would be foolish to *limitatively* enumerate the variables to be reckoned with. All we can hope for here is to portray some *canonical model* which could provide in most practical cases a sufficient (but not necessary) list of relevant types of variables. Although this was done in the session, we will skip here the description of the canonical model, which might lead the reader astray. We only refer the interested reader to an earlier (and hence less elaborate) version of it (Munier, 1994) and we immediately turn to the research strategies toward modeling bounded rationality models.

2. Research strategy

The development of any operational model implies that choices be made, which depend on the purpose of the model searched for. Identifiable research strategies in bounded rationality modeling fall into three broad categories: (i) a first strategy tries to develop complements to existing strong rationality models or to relax some of the hypotheses these models traditionally make; (ii) a second way to go consists in modeling the results of experimental observations, disregarding existing models; (iii) a last strategy looks for a compromise to the extent where either observed behavior 'modes' are used to simplify maximization problems, or, alternatively, simple adaptive rules are followed, but are in the first place derived from higher-level optimization.

We try to give examples of these three strategies in this section. How to relate each of these three strategies to the two categories of situations identified in our typology as the privileged realm of bounded rationality modeling is left to the reader until we offer some temporarily concluding remarks.

2.1. Relaxing and complementing strong rationality models

This first perspective keeps two different avenues open for research: (i) taking proxies as variables in the utility function, (ii) in the case of risk, developing the rank dependent and, more generally, non expected utility choice functionals.

2.1.1. Proxies which make agents smart. It may happen that economic agents rationalize, ex post, what they have done by finding an *ad hoc* utility function which yields, after maximization, the choice substantively made (cognitive dissonance can be looked at as an extreme variant of that category). In a similar way, one can show that if agents use proxies as guidelines for decision making, then boundedly rational choices may be interpreted as thoroughly rational decisions using the proxies. In other words, boundedly rational procedures can be reduced to agents choosing proxies to save information costs. Otherwise, the agents look for the best decision *on this basis*.

Take as an example the 'compromise effect' case (Simonson, 1989). Experimental results show that many people choose a bottle of wine A if presented with two bottles A and B, priced \$15 and \$20 respectively whereas, when presented with three bottles of wines A, B and C, priced respectively \$15, \$20 and \$30, they change their choice and take bottle B. This is a violation of the independence of irrelevant alternatives axiom (sometimes called also 'regularity' property). How can this example be accountable for by a 'strong rationality' model?

Wernerfelt (1995) argues that it can often be assumed that consumers have to rely only on their relative tastes (due to difficulties in evaluating attributes' weights, for example, and hence 'absolute' tastes) and that they take for proxies of these the market offerings, on the assumption that the latter reflect the scale of the different needs of the population. He calls this the 'rank-order rule' and shows then that the regularity assumption can be violated by consumers maximizing their expected utility. More specifically, if consumers are presented first with a small choice set and then with a large choice set (including the former one), they may change their choice as long as they are uncertain about how the elements of the small (initial) choice set rank in the larger set. Wernerfelt shows that this 'rank-order' rule is dynamically sustainable on a market with consumers belonging to different segments and endowed with different information states and two firms playing a two stage game (quality, price). The Nash equilibrium on this market tends asymptotically to the one with fully informed players when the purchasing cycle's length reduces w.r.t. the sellers' horizon and, of course, when the proportion of boundedly informed players tends to zero.

This last point is especially interesting. While it is not out of reach to find proxies which, once taken as arguments of a utility function, lead to choices that depend on the possibility set (and hence to boundedly rational choices, see section 1), it is not frequent, indeed, that maximizing such a utility function leads to the rational equilibrium which would have been attained if all players had been acting in a strongly rational way.

2.1.2. Risk and the rank dependent preference functional. The rank dependent preference functional is a straightforward weakening of expected utility, based on initial intuition by Allais (1952) and effectively developed by Quiggin (1982, 1993). The change in the model consists in inserting a 'probability transformation function' Θ, defined on the decumulative probability of the variable in the utility function and monotonically mapping this decumulative probability into [0,1] with $\Theta(0) = 0$ and $\Theta(1) = 1$. Weber and Kirsner (1997) show that the reasons for such transformation encompass potential perceptual biases, individual dispositions ('pessimism' as distinct from risk aversion, e.g.), but also reactions to a specific decision situation.

An interesting case can be made that this specific relaxing of hypotheses (of the independence axiom, for that matter) is a rough substantive approximation of observable behavior, on the condition that the parameters of the model have to be adjusted to subsets of probability distributions ('risk structures'). This not only corroborates the results by Weber and Kirsner just mentioned, but also the Simonian idea that decisions result from procedures looking first at the type of problem faced ('recognition') and setting then into motion appropriate heuristics ('heuristics search'). It also confirms that there are modeling heuristics (here, the probability transformation function) which are good descriptors of behavior although they ignore the procedures underlying these boundedly rational choices (Munier, 1999). Clearly, then, rank dependent utility belongs to the present category of research strategies, going 'topdown' and relaxing strongly rational models to yield a substantive counterpart to procedurally rational behavior. For another and more detailed discussion of the rank-dependent model, the reader is referred to Baz et al. (1999).

2.2. Inference from experimental results

This second research strategy goes exactly the opposite way. It claims that one should, at least disregard, if not get rid of, existing strong rationality models and rely exclusively on experimental and empirical observations (maybe the very meaning of Simon's message).

This type of strategy has been developed by many psychologists, as is well known. Selten's recent research is part of such a research strategy in bounded rationality modeling (Selten, 1997).

At least three kinds of mental processes seem to emerge from the experiments considered by Selten (his own and others): (i) Motivation, (ii) Adaptation, (iii) Cognition (all reasoning processes).

Motivation is acknowledged here as the underlying driving force of human behavior. It encompasses what we aim at and how fears and desires, fairness and reciprocity, etc., combine to inspire our action. Adaptation is the process by which people, not unlike animals without reasoning capacity, 'learn' and adjust their behavior. Human behavior, however, does not only result from the interaction of motivation and adaptation. It is also deeply influenced by cognition. Cognition can be defined as featuring all reasoning processes of the human mind, whether conscious or unconscious.

2.2.1. Motivation. The driving force of our behavior is strongly affected by context effects. For example, the presentation of experimental two-person games (Pruitt, 1970) under a decomposed form instead of under the standard bimatrix form will affect observed equilibria of these games. This may be related to the importance of perceived reciprocity or fairness in motivation. Results obtained by Winter and Zamir (1997), showing that the proportion of 'tough' players in an experiment has an impact on the motivation of each player confirm this interpretation. In the same way, experiments by Kahneman and Tversky (1979) and by Poumadère et al. (1995) can be interpreted as evidence that fear (or safety comfort) plays a role in behavior. Some authors would argue that this is still insufficient an insight and that the physical environment, leading to individual physiological effects and then to psychological processes, should be taken into account (Parker and Tavassoli, 1997).

Yet, solution concepts rely at least implicitly on motivation. It would be appropriate that they always explicitly rely on the motivation of each player. From an epistemological point of view, however, it seems that solution's concepts should avoid the very largely accepted character of self-reference. A self-referential concept of solution like the quota concept (determining the split of a coalition value between the members of the coalition) has considerably less predictive power than the equal division payoff bounds, which avoids any self-referential character. One can wonder whether these experimental results should not deter theorists from using self-referential concepts and incline to model step-by-step reasoning procedures.

2.2.2. Adaptation. Boundedly rational behavior often relies on ex-post rationality, contrary to the Bayesian point of view, which optimizes on the basis of current beliefs. Learning Direction Theory (Selten and Stoecker, 1986) describes a qualitative impact of cognition on adaptation, based on a world representation. The example of the archer (which aims more to the right if the preceding arrow went past the target to the left and conversely) is illustrative. Applications to bidding are in order. In this domain, the winner's curse 'paradox' occurs in so called common value auctions where the value of the object in the auction is not known to the bidders when they make their offers. Hence, if the estimated value is excessive, they get the object, but they lose money. If that value is not sufficiently reduced to avoid that risk, the winner of the auction (it is a first price sealed bid auction) makes a loss! The phenomenon can be explained as an application of ex-post local rationality (one could relate this finding to economizing behavior, see above). More precisely, however, Learning Direction Theory predicts changes in the right direction more frequently than randomly expected (tests at 5% significance level).

In the Bazerman-Samuelson (1983) bidding task, the value of an object to be sold is uniformly distributed over the integers $1, \ldots, 99$, for the seller. The value for the buyer is 50% higher. The latter knows the distribution of the value, but not the seller's value itself. The buyer has to name a price m and the object will be his if the price named is at least as high as the seller's value. If the buyer bids m, then in the cases in which he receives the object, the average value of the object for the seller will be $(m + 1)/2$. It will be 50% higher, that is $3(m + 1)/4$ for the buyer. But, for $m = 2, \ldots, 99$, this is smaller than m. It is therefore optimal for the buyer to bid 0 or 1. Nevertheless, the majority of the agents tested show no tendency towards the optimum (Ball, Bazerman and Carroll, 1991). If one tries to distinguish between the way subjects performed their task in such experiments, three categories of players emerge: (i) Analytical, (ii) Adapters and (iii) exceptional cases. Excluding the gambling addicts and non-participants of the last category, the percentage of analytical (gain maximizers, loss-avoiders or assets' conservers) was much smaller than the percentage of adapters. Adaptive behavior appears thus more frequent than highly rational maximizing behavior, on one hand; on another hand, statistical tests show that Learning Direction Theory performs much better for adapters than for other players (1% significance level, one-tailed).

2.2.3. Cognition and Reasoning. It is no wonder then that typical repeated games' experiments reveal neither highly rational behavior like optimization nor behavior based on quantitative expectations. Rather, fairness and efficiency considerations explain more of the cooperative or non cooperative behavior than sophisticated computations based on existing theories. Statistical analysis and careful debriefing of experiments support the idea that players do not form quantitative expectations about others' behavior. They instead try to exert an influence on their opponent's behavior. Strategies seem typically phased, with (i) a relatively short phase offering cooperation by announcing 'friendly' steps (in a duopoly, decreasing quantities put on the market) followed by (ii) a major phase of a tit-for-tat kind where reactions to other's steps are matched by own steps of similar size in the same direction, and finally (iii) a last phase where behavior becomes clearly non-cooperative. In this last phase, Cournot's solutions appear as descriptive of what happens (Selten, Mitzkewitz and Uhlich, 1997, Keser, 1997).

These appraisals of strategic behavior are by now insufficiently developed (especially with respect to motivation), so that it may appear difficult to design tests of refutability. On the other hand, one could claim that the question is not the one of refutation, but rather the one of relative performances of one theory with respect to others, closeness to data being here the appropriate measure to be taken into account.

2.3. Middle-of-the road strategies

As the preceding strategy is quite demanding in the sense where it is a long-term one, other efforts are in order, on which a quick account is given here. Such strategies are, on one hand, akin to the first strategy to the extent that they use maximizing standards, although in a certain restricted sense only. On another hand, they give a central role to observation and also to adaptive rules of the kind used by the second type of research strategy, described in 2.2.

Three subcategories emerge: (i) One can try, on an informal basis of general observation, to determine 'economizing modes of behavior'. These modes are interpreted as ways for the individual of overcoming or skirting the difficulty of the cognitive limitation. They are then modeled by using bits and pieces of 'strong' rationality models. (ii) A similar approach is taken by what can be subsumed under the broader category of 'evolutionary models', except that maximizing behavior is straightforwardly replaced by rule based behavior. Or (iii) one can interpret the adaptive rules used in boundedly rational procedures as resulting from a 'meta-rationality' level, the latter based on some type of maximizing behavior.

2.3.1. Economizing modes of behavior. It has for a long time been also noticed that agents do not make complex calculations prior to their choices (Knight, 1921). After all, computations use scarce resources and these need to be economized. As, however, incorporating decision costs into the decision problem may lead to infinite regress (Mongin and Walliser, 1988), it has been hypothesized that people, instead of explicitly using rational procedures taking decision costs into account, use what R. Day has called *economizing modes*. At least seven of such modes can be found (Day and Pingle, 1991). We give an account of these seven modes here:

(i) Trial and error search: Each action can be thought of as an experiment, its result can be compared to other actions' results and provide thus direction for the future search. Under relatively stable conditions, decisions are being improved and can approach optimality while the cost of decision making is being kept quite low.
(ii) Imitation: Mimicking another agent's decisions, even though it may not be that easy, is in general cheaper than finding a fully rational solution to one's own problem.
(iii) Following an authority: Looking at others' opinions or at others' actions is, as has already been stressed above, a fundamental input into preparing for a decision. However, we also can dispense with the pain of processing such informations and blindly follow the actions of those we consider — for better or for worse — as the experts to follow.
(iv) Habit: For similar reasons, we can decide that, for some specific problems, we are the expert to be followed, and we exercise thus routine purchases or other routine actions. This saves almost every task of thinking.
(v) Unmotivated search: Pure impulses, sometimes driven by more basic propensities, like the sense of adventure, etc., also provide radical savings on decision costs.

(vi) Acting according to hunch: Perhaps one way of giving an account of unmotivated search is to notice that actors when they do not find any rational support for whatever task they have to comply with, return to their basic 'mental equipment', to their basic intuitions (as an example, some politicians try to scratch more of government intervention, even if they don't have any rational argument for doing so, while some other decision makers look for more of the same, without any more rational background, etc.).

(vii) Procedural optimizing: A way to approach optimal solutions may consist in simplifying global problems by solving locally (using first order linear approximations) the issues raised and looking then for the direction of improvement (the gradient method of mathematical programming can be of help) and for the next issue which this behavior raises, which will, in turn, be solved locally, etc. This by no means amounts to an optimal search, but is rather a search for approximate optimality.

Experiments confirm existence of at least some of these modes (Pingle and Day, 1996). All seven of these modes of economizing can be modeled more or less precisely and possibly serve to accommodate existing models. An algorithm of 'cautious sub optimizing', mixing some general idea of risk aversion and the idea of imitation has been developed along the lines of (vii). This amounts to a weakened version of strong rationality behavior. Such models pay attention to several processes put forward by bounded rationality (Day, 1971). It should be stressed, however, that they show, in the first place, that using non procedurally rational processes may lead to approximations to full rationality. Indeed, quite paradoxically, if the costs economized are large as compared with the approximation obtained, economizing could lead to more favorable consequences than using a fully rational scheme of decision making, assuming the latter would be feasible!

2.3.2. Evolutionary Modeling. Evolutionary modeling focuses on three main concepts: the diversity of economic agents, in particular the internal organization of firms; the dynamics of relations between agents and their general environment, in particular technological change and innovation; the rule-based behavior of actors in the system. Originally derived from Schumpeterian economics, these models are bounded rationality ones and have implications for management practice and science. Slack management and business governance, the importance of marketing and distributional techniques are topics in which such modeling may have a comparative advantage (Tisdell, 1996, chs. 9, 16, 19). Slack management can be shown as better in some cases than complete optimization. As in the Wernerfelt model above, bounded rational behavior may provide an opportunity to improve upon the strongly rational equilibria. Arguments in that direction have also been suggested here and then (Munier, 1993, e.g.).

Quality revealing price is also a domain where evolutionary modeling has led to some interesting results. For example, to come back to an example in wine choice, evolutionary models show that in some cases, informed agents can influence prices in such a way that these prices will reveal to the whole market their information, i.e. such that every agent will learn to correctly guess the quality of each wine (Lesourne, 1991, ch. 6).

2.3.3. Two-level rationality modeling. This last type of strategy tends to view rule-based behavior as resulting from a compromise between exploration, a distinctive character of procedural rationality, and optimization, the distinctive character of substantive rationality.

An example has been given by Bourgine (1999) under the form of a model with a stopping rule. As has been already stressed above, incorporating decision costs explicitly in decision making leads to infinite regress and to more computations than in the standard case, and a stopping rule can hardly be derived in this way. If it can, it cannot be recognized as resulting from bounded rationality. On the other hand, arbitrary stopping rules do not solve the problem in a very satisfactory way. Such arbitrary stopping rules let us lose the property of duality (with all its economic significance) embodied in strong rationality models and do not offer any substitute. An idea is therefore to derive these stopping rules from some simple higher order principle, so that thresholds can recover some economic meaning. In this perspective, one can hope for thresholds modeled according to observation.

In such a framework, the stopping rule should be derived from what can be called the meta-problem (or higher order problem). This meta-problem will be intimately related to the information process of the agent. Hence, learning will be an essential process of the meta-problem. Typically, then, this kind of modeling will make use of two different levels: (i) the information system level and (ii) the evaluative system (some utility scheme can be used for that purpose).

As an example, let us consider a sequential decision problem and denote the state of information at time t by F_t. At each time t, the consumer has to choose between two products, the quality of which are identically independently distributed random variables. However, the consumer (under an uncertainty of order two) doesn't know unambiguously these distributions. Assume, for example, that the consumer has tried product one three times and product two forty times. She has found product one of high quality once and product two twenty times. Which one should she try next time? Clearly, the successful use frequency of product two (50%) is higher than the one of product one (33%). But, on the other hand, she used product one much less frequently than product two. The idea is therefore that the design of the rule telling her when to change products should depend on a compromise between exploring more of product one effects and exploiting product two's frequency of success. Say, the distributions of the quality of each of these two products can be of three different types. Assume the consumer has beliefs on each of these states. At each time t, the consumer revises her earlier beliefs F_{t-1} and turn them into F_t (Bayes' rule is mostly used, but this might not be essential). Then, maximizing some intertemporal (additive over time) expected utility on the basis of F_t can yield a simple decision rule (of a threshold type, hence comparable to a 'satisficing' rule) which behavior should obey. Gittins indices are the best example (Gittins, 1989), but Bourgine gave at the session a different example, based on Snell's envelope.

Generalizing this kind of reasoning can lead to a ('metasatisficing') theory of satisficing solutions. The real modeling difficulty is to categorize what, in the past, matters for the present. Now, whatever is relevant in this respect for the individual provides *a theory of where the 'satisficing' threshold comes from*: we can thus discuss it in a precise way and possibly derive its meaning, which might be more difficult to do in many cases of the second research strategy evoked above. One difficulty for these models can be that of multidimensionality (important in marketing) if there is no separability in the multi-attribute utility function (the arguments of Ut are not identically distributed over time and independently distributed from each other). But these assumptions hold in quite a few cases. This kind of model can then be useful to describe how people adjust to live with limited computational capacities in a world of almost infinite complexity: they select in a sophisticated way simple rules on which to base their behavior.

3. Summary and concluding remarks

Bounded rationality modeling is becoming an important part of economic analysis as well as of marketing science. We have discussed the reasons for this research program and suggested a typology of situations to help us recognize when and in what respect boundedly rational modeling can be deemed to be particularly useful. We have finally described the three main types of research approaches which are being used and can be used in the research ahead.

In the present state of the art, these three strategies of research are rather complementary approaches than substitutes. A priori, it seems that strategies described in 2.1 and in 2.3 would be better fitted to deal with situations of type 2 in our typology whereas the strategy described in 2.2 seems rather fitted to deal with the more complex situations of type 3 in our typology. All will help understanding what rationality is about and figuring out the behavior of economic agents. According to the needs, a researcher might thus be willing to choose either one of the three approaches. From an epistemological point of view, however, we need to compare and meaningfully evaluate alternative models. We have stressed that a meaningful evaluation requires a minimal consensus about what kind of situation has to be explained.

Clearly, if we want to remain as closely as possible not only to available evidence, but also to data on some specific situations, experimenting on rational behavior will be an inescapable avenue for research. Two remarks need however to be added here:

1. Consumer behavior will remain, in marketing, the major field in which such experiments will develop. In this respect, joint research with experimental economists should help making protocols more complete and conclusions perhaps more robust. This cooperation should be in the future as successful as has been the cooperation with psychologists, by now a long tradition already.
2. We badly need experimental studies of business behavior as well as of public economic behavior. To do this, saliently realistic protocols have to be found and implemented on computers before we can observe through live experiments what has to be learned in these areas.

Progress of knowledge in marketing as well as in economics needs that the price be paid in the first place.

References

Abdellaoui, M. and B. Munier (1994) On the Fundamental Risk-Structure Dependence of Individual Preferences under Risk: An Experimental Investigation, *NR GRID 94–07*. Ecole Normale Supérieure de Cachan.

Abdellaoui, M. and B. Munier (1998a) The Risk-Structure Dependence Effect: Experimenting with an Eye to Decision-Aid, *Annals of Operations Research,* **80**, 237–252.

Abdellaoui, M. et B. Munier (1998b) Testing Probability tradeoff Consistency in Individual Decision Making, NR GRID n° 98–13, mimeo, Ecole Normale Supérieure de Cachan.

Abdellaoui, M. and B. Munier (1999) How consistent are Probability tradeoffs in individual preferences under risk? Some preliminary results, in: Machina, M. J. and B. Munier, eds., *Beliefs, Interaction and Preferences in Decision Making*, Dordrecht/Boston, Kluwer Academic Publishers. (forthcoming).

Allais, M. (1952). Fondements d'une théorie positive des choix comportant un risque, *Econométrie*, Colloques Internationaux du C. N. R. S., Paris, Publications du CNRS. 127–140.

Allais, M. (1974), *Contributions à la science économique 1943–1974*, mimeo, Centre d'Analyse Economique N° 36–03, Ecole des Mines de Paris.

Armstrong, J. S. (1985) *Long Range Forecasting: From Crystall Ball to Computer*, New York, Wiley.

Ball, S. B., M. H. Bazerman and J. S. Carroll (1991) An Evaluation of Learning in the Bilateral Winner's Curse, *Organizational Behavior and Human Decision Processes*, **48**, 1–22.

Bazerman, M. H. and W. Samuelson (1983) The Winner's Curse: An Empirical Investigation, in: R. Tietz (ed.), *Aspiration Levels in Bargaining and Economic Decision Making*, Lecture Notes in Economics and Mathematical Systems, N° 213, Berlin/Heidelberg, Springer Verlag.

Baz, Jamil, et al. (1999), Risk Perception in the Short Run and in the Long Run, *Marketing Letters*, this issue.

Becker, J. and R. Sarin, 1987, Lottery-Dependent Utility, *Management Science*, **11**, 1367–1382.

Bolger, F. and N. Harvey (1998) Heuristics and Biases in Judgmental Forecasting in: G. Wright and P. Goodwin, (eds.), *Forecasting with Judgment*, New York, Wiley.

Conlisk, J. (1996) Why Bounded Rationality, *Journal of Economic Literature*, **36**, 669–700.

Day, R. H. (1971) Rational Choice and Economic Behavior, *Theory and Decision*, **1**, 229–251.

Day, R. H. and M. Pingle (1991) Economizing economizing, in: R. Franz, H. Singh and J. Gerber, eds., *Handbook of Behavioral Economics*, 2B, Greenwich, Connecticut, JAI Press, pp. 511–524.

Gittins, J. C. (1989) *Multi-armed Bandit Allocation Indices*, New York, Wiley.

Harvey, N. (1999) Improving Judgment in Forecasting, in: J. S. Armstrong (ed.), *Principles of Forecasting: A Handbook for Researchers and Practitioners*, Norwell, Mass., Kluwer Academic Publishers. (forthcoming).

Kahneman, D. and A. Tversky (1979) Prospect Theory: An Analysis of Decision under Risk, *Econometrica*, **47**, 263–291.

Kampmann, C. and J. D. Sterman (1998) Feedback Complexity, Bounded Rationality and Market Dynamics, Sloan School of Management, M. I. T. Mimeo.

Keser, C. (1997) *SUPER: Strategies used in Public Goods Experimentation Rounds*, Discussion Paper, University of Karlsruhe.

Knight, F. (1921) *Risk, Uncertainty and Profit*, Chicago, Chicago University Press.

Kreps, D. M. (1997) Bounded Rationality in: P. Newman (ed.), *The New Palgrave Dictionary of Economics and the Law*, London, Macmillan. P. 168–173.

Lesourne, J., 1991, *Économie de l'ordre et du désordre,* Paris, Economica.

Machina, M. J. (1989) Dynamic Consistency and Non-Expected Utility Models of Choice Under Uncertainty, *Journal of Economic Literature*, **XXVII**, 1622–1668.

Mongin, P. (1994) L'optimisation est-elle un critère de rationalité individuelle?, *Dialogue*, **33**, 191–222.

Munier, B. (1993) Komplexität, das Konzept Unsicherheit and beschränkte Rationalität des Menschen, *Zeitschrift für Wissenschaftsforschung*, **7–8,** 113–126.

Munier, B. (1994) Décision et Cognition: Quelques points de repère, *Revue Francaise de Gestion*, **99**, 79–91.

Munier, B. (1999) Two-Stage Rationality Under Risk: Experimental Results and Perspectives, *Rivista di Matematica per le Scienze Economiche e Soziali*, forthcoming

Parker, P. and N. T. Tavassoli (1997) Physioeconomic Theories of Culture and Consumption, WP 97/70/MKT, INSEAD, Fontainebleau, France. Mimeo.

Poumadère, M., C. Mays, P. Slovic, J. Flynn and S. Johnson (1995) What lies behind Public Acceptance? Comparision of U. S. and French Perceptions of the Nuclear Power Option, *Proceedings of an International Conference on the Nuclear Power Option*, Vienna, International Atomic Energy Agency.

Pingle, M. and R. H. Day (1996) Modes of Economizing Behavior: Experimental Evidence, *Journal of Economic Behavior and Organization*, **29**, 191–209.

Prelec, D., B. Wernerfelt and F. Zettelmeyer (1997) The Role of Inference in Context Effects: Inferring what You Want from What is Available, *Journal of Consumer research*, **24**, 118–125.

Pruitt, D. G. (1970) Reward Structure of Cooperation: The Decomposed Prisoner's Dilemma Game, *Journal of Personality and Social Psychology*, **7**, 21–27.

Quiggin, J. (1982) A Theory of Anticipated Utility, *Journal of Economic Behavior and Organization*, **3**, 4, 323–343.

Quiggin, J. (1993) *Generalized Expected Utility Theory, The Rank-Dependent Model*, Boston/Dordrecht, Kluwer Academic Publishers.

Rubinstein, A. (1998) *Modeling Bounded Rationality*, Cambridge, Mass., M. I. T. Press.

Savage, L. J. (1954) *The Foundations of Statistics*, New York, Wiley.

Selten, R. (1987) Equity and Coalition Bargaining in Experimental Three-Person Games, in: A. E. Roth, ed., *Laboratory Experimentation in Economics, Six Points of View*, Cambridge, Cambridge University Press. Pp. 42–98.

Selten, R. (1997) Features of Experimentally Observed Bounded Rationality, *DP B-421, Sonderforschungsbereich 303*, University of REF.

Selten, R., M. Mitzkewitz and G. R. Uhlich (1997) Duopoly Strategies Programmed by Experienced Players, *Econometrica*, **65**, n°3, 517–555.

Selten, R. and R. Stoecker (1986) End Behavior in Sequences of Finite Prisoner's Dilemma Supergames, *Journal of Economic Behavior and Organizations*, **7**, 47–70.

Simon, H. A. (1986) Rationality in Psychology and Economics, in: R. M. Hogarth and M. W. Reder (eds.), *Rational Choice*, Chicago, The University of Chicago Press. 25–40.

Simon, H. A. (1995) The Information-Processing Theory of Mind, *The American Psychologist*, **50**, 7, 507–508.

Simonson, I. (1989), Choice Based on Reasons: The Case of Attraction and Compromise Effects, *Journal of Consumer Research,* **16**, 158–174.

Smith, V. L. (1991) Rational choice: The contrast between economics and psychology, *Journal of Political Economy*, **99**, 4, 877–897.

Snell, J. L. (1952) Applications of martingale Systems Theorems, *Trans. Am. Math. Soc.*, **73**, 293–312.

Tisdell, C., 1996, *Bounded Rationality and Economic Evolution, A Contribution To Decision Making, Economics and Management*, Cheltenham/Brookfield, Edward Elgar.

Weber, E. U. and B. Kirsner, Reasons for Rank-Dependent Utility Evaluation, *Journal of Risk and Uncertainty*, **14**, 41–61.

Wernerfelt, B. (1995) A Rational Reconstruction of the Compromise Effect: Using Market Data to Infer Utilities, *Journal of Consumer Research*, **21**, 627–633.

Winter, E. and S. Zamir (1997) An Experiment with Ultimatum Bargaining in a Changing Environment, DP 159, December, Center for Rationality, The Hebrew University of Jerusalem.