



Notes and comment

On the coefficient of variation as a predictor of risk sensitivity: Behavioral and neural evidence for the relative encoding of outcome variability

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ABSTRACT

Cox and Vjollca (2010) argue that the coefficient of variation (CV) fails as a normative criterion for risk taking in risky choice because it predicts violations of dominance. They suggest that it fails also on descriptive grounds because such violations are not observed in a study they conducted and because people's choices in two other situations were not predicted by the CV. This paper counters the normative argument by suggesting that occasional violations of dominance might be the price that organisms with only limited processing capacity pay to achieve a broad set of goals. The consistency axioms of rational choice theory have a long history of falling short of accounting for such tradeoffs. The paper then addresses the three instances of descriptive failures of the CV to predict risk taking provided by Cox and Sadiraj, showing them to be inappropriate or inconclusive arguments against the use of the CV as a measure of risk in risk-return models of human and animal risk taking. Finally, the paper reviews new behavioral and neuroscience evidence in support of the CV as a predictor of risk taking, especially in decisions from experience, for which the model was developed.

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Axiomatic choice models, like von Neumann and Morgenstern's (1947) expected utility (EU) theory, impress with the Platonic beauty of eternal truth. Consistency in preference across different choice sets is a compelling normative requirement. Who would not want her choices to be transitive and thus avoid being turned into a money pump, thus making EU's axioms also reasonable contenders to describe risky choice? Together with transitivity, dominance of preference is a convincing axiom. We all like to get something for nothing, and thus choose the option that offers everything other options do and more.

Early psychological investigators of the EU model (e.g. Edwards, 1954) were fully expected to find empirical support for its axioms, and probably were as surprised as economists by the systematic violations of some axioms and deviations of choices from the predictions of the EU model. These deviations generated a large industry in alternative risky choice models, which have been extensively reviewed elsewhere (e.g., Starmer, 2000). Prospect theory (PT) emerged as the most prominent generalization of EU that explains both EU's predictive successes and its failures. Daniel Kahneman received the 2002 Nobel Prize in Economic Science for his development of PT in collaboration with Amos Tversky. In light of Cox and Sadiraj's (Cox & Vjollca, 2010) concern about the normative status of the coefficient of variation (CV) as a predictor of risk sensitivity because it allows for possible dominance violations, it is worth noting that the original version of

PT (Kahneman & Tversky, 1979) also predicted violations of first-order stochastic dominance, which prompted a reformalization of its decision weight function in the cumulative PT version (Tversky & Kahneman, 1992). This reformalization left PT's basic predictions intact, but removed the property of dominance violations so bothersome to many economists. It is conceivable that the psychophysical CV–EV version within the risk-return framework proposed by Weber, Shafir, and Blais (2004) might be reformulated in a way that would preserve the features that give it its predictive power, but would remove its possibility of occasionally predicting the choice of dominated alternatives. As we will see below however, dominated preferences are in fact observed on occasion.

Risk-return models of risky choice take as their primitives not the outcomes of risky choice options and their likelihoods, but moments of the distribution of outcomes. Normative risk-return models that predict risk taking based on the first and second moment (i.e., EV and Variance) have been formulated both in finance (Markowitz, 1959) and in evolutionary biology (Caraco, 1980). Psychophysical risk-return variants on these models (Sarin & Weber, 1993; Weber & Hsee, 1998; Weber & Milliman, 1997) attempt to do for the normative Variance–EV model what PT did for EU. Models like the CV–EV version proposed by Weber et al. (2004) try to explain systematic deviations in observed levels of risk taking by people and other species from the predictions of normative models, by using 150 years of psychological insights into attentional and perceptual processes (Weber & Johnson, 2009). People share basic perceptual, encoding, and associative processes with other animals, but also have evolved to employ

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abstract, symbolic representations and the ability to manipulate and communicate such representations, abilities not found in lower animals. Comparative studies that compare what people vs. song birds or honey bees do in risky foraging situations can be instructive both about the similarities and the differences in behavior.

One important recent distinction that has resulted from such studies is that between (risky) decisions made from experience vs. from description (Hertwig, Barron, Weber, & Erev, 2004). In *decisions from experience*, people, birds, and bees find out about the different outcomes of available choice alternatives by repeatedly sampling them and experiencing their consequences. Positive consequences increase the likelihood that the option is chosen again, whereas negative consequences decrease it. *Decisions from description* are available only to human decision makers, i.e., only people can understand and process summaries about the outcomes of different choice options and their likelihood, communicated to them in graphic or numeric form, as in the prospectus of an investment fund or the pie charts that describe different choice options in an experiment. When small probability events are involved, people's choices can differ drastically when decisions are made either from experience or from description (Weber et al., WSB, 2004). PT was developed to account for choices made under description and predicts that rare events tend to be overweighted, relative to their likelihood of occurrence. Weber et al. (2004) showed that the reinforcement learning models that predict decisions from experience tend to underweight rare events. In a direct comparison of decisions under the two learning conditions, they found that the CV was a vastly better predictor of risk taking in decisions from experience for both people and other animals, but was at best marginally better than the variance or standard deviation in decisions from description, for which PT provides good predictions. They showed that associative learning of which choice options result in better outcomes leads to choices that are associated with the CV, and not the variance of outcomes. C&S acknowledge the fact that the CV was proposed by Weber et al. (2004) as a predictor of risk taking in decisions from experience, yet the three studies presented as evidence for the descriptive failures of the CV ask respondents to make decisions from description.

In the remainder of this paper, I first address C&S's normative concern. I then suggest reasons for C&S's failure to find levels of risk-taking predicted by the CV in the three studies they describe, above and beyond the important general point that the CV–EV model was not primarily proposed to explain decisions from description. I end by reviewing recent behavioral and neuroscience evidence that suggests that the CV does indeed provide a useful proxy for perceptions of risk that, in combination with other characteristics of risky choice options (such as their EV and type and degree of skew) predicts risk taking well, especially in decisions from experience.

1. Normative status of CV as index of risk and C&S's Experiment 1

People have a broad range of goals, some of which can be conflicting. We observe goal conflicts frequently enough to have sayings about people who “want to have their cake and eat it, too”. Consistency of behavior, including choices, is an important goal. Cialdini (2001) has described, for example, how the desire for consistency can be preyed on in persuasion techniques. However, we also have other goals that get activated by different choice contexts and can thus lead to inconsistencies of preference. The insight that preferences are often constructed online when people are asked to make a decision, rather than simply expressed, may well be psychology's most successful export to economics (Weber & Johnson, 2009). Inconsistencies in preference large

enough to result in systematic preference reversals have been reported by psychologists (see Lichtenstein & Slovic, 2006) as well as economists (Cubitt, Munro, & Starmer, 2004; Grether & Plott, 1979). When people are confronted with such inconsistencies in their preferences across choice situations, they sometimes are willing to revise one or the other of their original preferences to restore consistency, e.g., in the case of intransitive choice patterns, acknowledging that the differential attention paid to different option characteristics or choice goals was, in fact, an error. At other times, however, as in the famous account by Samuelson (1963), people are “content to satisfy their preferences and let the consistency axioms satisfy themselves,” thus confirming the legitimacy of applying different goals or criteria to different choice situations.

This gets us to the question about the normative status of dominance violations. There is no question that, when people are confronted with transparently dominated alternatives, as in Choice J vs. K in C&S's Experiment 1, they will not choose such options. Choosing dominated alternatives is so patently irrational that such choices are often included as “catch trials” in decision experiments to identify and eliminate respondents who are not paying attention to task instructions and might be responding randomly, or who might be brain damaged. For choices like the one between Option J and K in C&S's Experiment 1, people use very simple rules (“\$74 is better than \$20”), that have nothing to do with either CV–EV based choice or EU or PT theory. In this sense, C&S's Experiment 1 tells us nothing new about the descriptive fit of the EV–CV model.

We do, however, observe violations of dominance when the fact that such violations are committed is not apparent to the decision makers, e.g., in studies that use between-subject designs and ask subjects to price risky options. Such implicit dominance violations seem to be the result of neural (perceptual or psychophysical) processes that have to do with the relative (and comparative) encoding of magnitudes. People will pay less for lottery A: (a 7/36 chance to win \$9; otherwise win 5¢) than for lottery B: (a 7/36 chance to win \$9; otherwise win nothing), and less for lottery B than for lottery C: (7/36 win \$9; otherwise lose 5¢). A large positive payoff seems more impressive when it occurs with (and thus is being compared to) another outcome that is 0 than with a small positive amount (Birnbaum & Thompson, 1996), and even more when it is paired with (and thus compared to) a small negative amount (Bateman, Dent, Peters, Slovic, & Starmer, 2007). These effects have been observed in multiple studies and are very reliable. At the same time, there is no question which of these three lotteries people would choose in a direct comparison, when their violation of dominance would be obvious.

While dominance violations clearly hold no normative appeal, it may still be true that a normative argument could be made for the CV rather than the variance as summary index of variability or risk. Evolutionary optimization may not adhere to the strict consistency requirements of economic rationality, and the occasional inconsistency (like a dominance violation) might be a small price to pay for reduced processing loads and other advantages brought by decision criteria that might lead to such inconsistencies. Judgments of how normative (in the sense of optimal or adaptive) a decision rule like the CV is, requires knowledge of the characteristics of the environment in which the organism has to function. Shafir and Weber have suggested that the relative encoding of variability provided by the CV is adaptive in light of the frequency distributions of objects (like mean nectar amounts found in different foraging patches or the population size of US cities) between which bees and people have to discriminate in their ecological environments. In a nutshell, if an organism would like to be able to maintain similar discriminability between objects whose distribution function of magnitude has the inverse J-shaped, highly skewed form described by Zipf (1949) for a wide

Table 1

Characteristics of option pairs described as Choice Tables Q to T in C&S Experiment 2. Each option pair contains one sure option and one two-outcome option of equal expected (EV). These two-outcome risky options in the six option pairs differ only somewhat in EV, but significantly in variance (and thus standard deviation and the coefficient of variation, $CV_{100} = (STD/EV) * 100$), and also in the direction and their degree of skewness.

Option pair/choice table	Q	S	U	W	R	T
EV	7	27	35	31	22	15
Variance	3	262	992	1,883	1,482	1,406
STD	1.7	16.2	31.5	43.4	38.5	37.5
CV_{100}	25	60	90	140	175	250
Skew	–	–	0	+	++	+++

range of linguistic, sociological, biological, physical, and economic phenomena, it makes sense to encode size in a way that gives rise to JNDs that follow Weber's (1834) law and to perceptions of variability that are approximated by the CV (see Shafir, Bechar, and Weber (2003) and Weber et al. (2004) for details).

2. C&S's Experiment 2

In their second test of the descriptive fit of the CV model, C&S use a series of choices between two-outcome lotteries and a sure thing of equal EV. These are choices from description, i.e., participants get told what the possible outcomes and their likelihood of occurrence are, by verbally describing to them how the probabilities of different outcomes are determined by different numbers of high and low payoff balls in an urn. These urns are completely hypothetical and only are used to describe outcome probabilities, as shown in C&S's Table 1. Each option pair contains one sure option and one two-outcome lottery of equal expected (EV). These two-outcome risky options in the six option pairs differ only somewhat in EV, but differ quite significantly in variance, and thus also standard deviation and the coefficient of variation, $CV_{100} = (STD/EV) * 100$, as depicted in C&S's Figure 1. As summarized in this paper's Table 1 though not pointed out in C&S, the two-outcome lotteries also differ very greatly in the direction and degree of skew.

C&S acknowledge that the CV model of risk sensitivity was proposed to predict decisions from experience, but then decide to go ahead and test it for decisions from description, without subsequently qualifying their conclusions about the model's descriptive failures. They claim to refute the model because the proportions with which their respondents choose the sure option in these choices do not track the increasing CV across the six choice pairs, but instead hover between .5 and .6, without any noticeable increase. This claim reflects two misconceptions about the CV risk sensitivity model of Weber, Shafir, and Blais (Weber et al., 2004). First, WSB never suggest that the CV is the *only* predictor of risk taking. In their meta-analysis of the large number of existing studies that examined choices of exactly the type used by C&S in their Experiment 1, between a two-outcome lottery and a sure option of equal expected value, they identified a multitude of other factors (including the skew of the lottery outcomes) that had large effects on risk taking. Elsewhere (e.g. Luce & Weber, 1986) I have argued that perceptions of variability or risk also exhibit different sensitivity to upside or downside variation, which should thus receive different weights in measures of risk. All that WSB suggested was that the CV, as a relative measure of variability per unit of return, provides a better predictor of risk taking than an unstandardized measure like the variance of outcomes. It is perhaps ironic that C&S's data from Experiment 1 actually confirm this claim. As Table 1 shows, the variance of lottery outcomes increases, not quite monotonically but quite dramatically, from choice pair Q to T, by a factor of 750, compared to the increase in CV by a factor of 10. A regression of the proportion of respondents

selecting the sure option reported by C&S on either the CV or the variance shows that neither CV nor variance predict the relatively stable level of risk taking across these choice pairs very well. However, the CV accounts for twice as much variance in choice probabilities (12%) than the variance (6%), and so the variance does a worse job than the CV. One way of interpreting the results of C&S's Experiment 2, is to observe that they call into question the descriptive fit of the Capital Asset Pricing Model (CAPM), the leading model for pricing risky assets in finance, which essentially trades off between asset EV and variance. This model has a lot of normative justification, but like the EV–CV model, does not incorporate any skewness preference, and hence fails to predict for this set of risky choice, in which skew varies strongly across choice pairs in ways that offset the effect of the CV or variance. So would C&S argue that we should get rid of the CAPM?

Other researchers have shown the influence of choice option characteristics above and beyond the CV on risk taking, described in Shafir et al. (2003). Drezner-Levy and Shafir (2007), for example, found that the skewness of the outcomes of risky foraging options and the presence of zero rewards affected risk sensitivity in honeybees, in addition to the CV. Marsh and Kacelnik (2002) report that risk sensitivity in birds depends on whether they perceive the choice options as relative gains or costs. In this context, C&S's Experiment 2 provides just another, not particularly novel illustration of the fact that the CV is not the sole determinant of risk taking.

3. C&S's Experiment 3

Table 2 describes the six choice pairs presented by C&S to respondents as their Experiment 3. Table 3 provides summary information about them. (Ignore Choice Pair 0 in both tables for now.) Each option pair contains one sure option and one 50:50 two-outcome lottery option, whose EV is \$5 more than the sure amount. The standard deviation of the lottery option is the same across all six choice pairs, though the EVs increase from \$3005 to \$110,005, which has implications for the coefficient of variation, $CV_{100} = (STD/EV) * 100$. C&S plot the CV as sharply decreasing in their Figure 2, at least for the first three choice pairs, but this plot is quite misleading in that the actual value of the CV is extremely small for all six choice pairs. (Compare the CV_{100} values of the choice pairs in Table 3 (with a maximum of 3 and a minimum of .1) to those in Table 1, where the CV_{100} values ranged from 25 to 250.) Closer inspection of the choice options that respondents faced in Experiment 3 (as shown in Table 2) confirms that the "lotteries" offered in those pairs did not differ much at all from the sure thing. In Choice Pair 3 for example, the choice is between \$50,000 for sure or a 50:50 lottery that pays either \$49,900 or \$50,110. Common sense as well as the CV–EV model of risk sensitivity predict that people ought to be pretty much indifferent between such choice options, which is what C&S observe, as described in the last row of Table 3. The fact that respondents somewhat favor the "lotteries" over the sure option for Choice Pairs 1 and 2 makes sense in light of the fact that the \$5 advantage in EV for the lottery over the sure option is more noticeable to respondents when outcome values are lower (i.e., the EV-advantage of the lottery is also encoded in relative, rather than absolute terms).

The following thought experiment will show that, rather than discredit the CV–EV model as a descriptive model of risk sensitivity, the basic design of C&S's Experiment 3 actually provides additional support for it. Let us extend the "recipe" for the creation of choice pairs (sure option pays \$x; 50:50 lottery pays either \$x – 100 or \$x + 110) to lower values of x (like Choice Pair 0 in Tables 2 and 3), that result in a value for the CV sufficiently different from zero to affect risk preference. For such choices, I would put a lot of money on the prediction that people's probability of choosing the sure option will be significantly greater than .5 and that they would increase with the size of the CV.

Table 2

Choice options presented to respondents as Choice Pairs 1–6 in C&S Experiment 3, and additional hypothetical choice pair that has the same design characteristics as C&S's choice pairs 1–6.

Choice pair	Sure thing	vs.	50:50 lottery
0	\$0		(\$-100; \$110)
1	\$3000		(\$2900; \$3110)
2	\$9000		(\$8900; \$9110)
3	\$50,000		(\$49,900; \$50,110)
4	\$70,000		(\$69,900; \$70,110)
5	\$90,000		(\$89,900; \$90,110)
6	\$110,000		(\$109,900; \$110,110)

Table 3

Characteristics of option pairs described as Choice Pairs 1 to 6 in C&S Experiment 3. Choice Pair 0 is a hypothetical additional choice pair that has the same design characteristics as C&S's choice pairs 1–6. Each option pair contains one sure option with EV_{sure} and one 50:50 two-outcome lottery option with EV_{lottery} . The standard deviation of the lottery option is the same across all six choice pairs, though the EVs differ, resulting in differences in the coefficient of variation, $CV_{100} = (STD/EV) * 100$. Note however, that all CV_{100} values are extremely small, compared to differences in CV_{100} values across the choice pairs in C&S Experiment 2, shown in Table 1. p (sure thing) shows the proportion of respondents choosing the sure thing in each choice pair.

Choice pair	0	1	2	3	4	5	6
EV_{sure}	0	3000	9000	50,000	70,000	90,000	110,000
EV_{lottery}	5	3005	9005	50,005	70,005	90,005	110,005
STD	105	105	105	105	105	105	105
CV_{100}	210	3	1	.2	.1	.1	.1
p (sure thing)	?	.41	.40	.48	.50	.54	.53

4. New behavioral and neuroscience evidence

Since 2004, additional empirical evidence for the superior ability of the CV (over the variance of outcomes, as suggested by biological normative models like Caraco's (1980) energy budget rule and finance normative models like CAPM) to predict risky (foraging) decisions has been provided from several sources. As already mentioned, Drezner-Levy and Shafir (2007) found that the CV predicts risk taking in combination with other option characteristics like skew, and review several other animal studies with similar results. Drezner-Levy, Smith, and Shafir (2009) found that the CV was a good predictor of risk sensitivity in bees with different foraging specializations. D'Acromont and Bossaert (2008) review evidence that the brain provides risk-return representations of risky choice options, just as it represents outcomes amounts and their likelihood. McCoy and Platt (2005) report evidence from single cell recordings in the posterior cingulate cortex of macaque monkeys that suggests that representations of risk are better described by a reference-dependent index like the CV than an absolute index like the variance.

Postulating that the CV rather than the variance of risky choice option outcomes ought to predict risk taking rests on perceptual and ultimately neural argument related to relative encoding of magnitude and changes in magnitude (Weber, 2004), long ago formalized in Weber's law (1834). Other examples of such psychological and perceptual (and not rational economic) arguments, essentially related to the detectability or discriminability of magnitudes or changes in magnitude can be found in Shafir, Reich, Tsur, Erev, and Lotem (2008). In a slightly different context, namely economic games, both orangutans (Furlong, 2008) and humans (Furlong & Opfer, 2009) were more likely to cooperate in a prisoners' dilemma situation when identical payoffs were shown to them in a smaller currency (pennies vs. dollars for humans, or grapes cut into small pieces vs. left intact for the apes), presumably because the larger magnitude of the payoffs for defection is less noticeable when expressed in pennies than in dollars. Similar currency-related differences in perceived magnitudes of identical economic payoffs (the

money illusion) have been shown to have a neural basis, e.g., can be found in the activation in reward-related brain areas like the ventromedial prefrontal cortex (vmPFC), associated with the processing of anticipatory and experienced rewards, and the valuation of goods (Weber, Rangel, Wibral, & Falk, 2009).

5. Conclusion

C&S critique a paper by Weber et al. (2004) (WSB) in two ways. They critically reexamine the WSB-proposed measure of the coefficient of variation (CV) as either a normative or descriptive criterion for decisions under risk and report that it fails as both. The present paper shows that the normative status of the CV as an index of risk is up for debate, but that the question is far from settled, given a broader conceptualization of rationality as adaptation to one's native environment and the decision makers processing constraints. The paper also shows that C&S's claims about the descriptive failures of the CV model are unpersuasive, provides alternative explanations for observed choice patterns, and identifies C&S misconceptions about the claims and applicability of the CV model. It seems odd and perhaps even misleading to explicitly make the point that the EV–CV risk-return model was not proposed to account for decisions from description, but then to proceed as if it did, by examining decisions from description in three different experiments. In addition, each of the three studies can be understood in ways that show observed choice patterns to be either irrelevant or supportive of the CV model of risk sensitivity.

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References

- Bateman, I., Dent, S., Peters, E., Slovic, P., & Starmer, C. (2007). The affect heuristic and the attractiveness of simple gambles. *Journal of Behavioral Decision Making*, 20, 365–380.
- Birnbaum, M. H., & Thompson, L. A. (1996). Violations of monotonicity in choices between gambles and certain cash. *American Journal of Psychology*, 109, 501–523.
- Caraco, T. (1980). On foraging time allocation in a stochastic environment. *Ecology*, 61, 119–128.
- Cialdini, R. B. (2001). The science of persuasion. *Scientific American*, 284, 76–81.
- Cox, J. C., & Vjollca, S. (2010). On the coefficient of variation as a criterion for decision under risk. *Journal of Mathematical Psychology*.
- Cubitt, R. P., Munro, A., & Starmer, C. (2004). Testing explanations of preference reversal. *Economic Journal*, 114, 709–726.
- D'Acromont, M., & Bossaert, P. (2008). Neurobiological studies of risk assessment: a comparison of expected utility and mean–variance approaches. *Cognitive, Affective, and Behavioral Neuroscience*, 8, 363–374.
- Drezner-Levy, T., & Shafir, S. (2007). Parameters of variable reward distributions that affect risk sensitivity of honey bees. *Journal of Experimental Biology*, 210, 269–277.
- Drezner-Levy, T., Smith, B. H., & Shafir, S. (2009). The effect of foraging specialization on various learning tasks in the honey bee. *Behavioral Ecology and Sociobiology*, 64, 135–148.
- Edwards, W. (1954). The theory of decision making. *Psychological Bulletin*, 41, 380–417.
- Furlong, E. E. (2008). Number cognition and cooperation. Ph.D. dissertation. Department of Psychology, The Ohio State University.
- Furlong, E. E., & Opfer, J. E. (2009). Cognitive constraints on how economic rewards affect cooperation. *Psychological Science*, 20, 11–16.
- Grether, D. M., & Plott, C. R. (1979). Economic theory of choice and the preference reversal phenomenon. *American Economic Review*, 69, 623–638.
- Hertwig, R., Barron, G., Weber, E. U., & Erev, I. (2004). Decisions from experience and the effect of rare events. *Psychological Science*, 15, 534–539.
- Kahneman, D., & Tversky, A. (1979). Prospect theory: an analysis of decision under risk. *Econometrica*, 47, 263–291.
- Luce, R. D., & Weber, E. U. (1986). An axiomatic theory of conjoint, expected risk. *Journal of Mathematical Psychology*, 30, 188–205.
- Lichtenstein, S., & Slovic, P. (2006). *The construction of preference*. London: Cambridge University Press.
- Markowitz, H. M. (1959). *Portfolio selection*. New York: Wiley.
- Marsh, B., & Kacelnik, A. (2002). Framing effects and risky decisions in starlings. *Proceedings of the National Academy of Science*, 99, 3352–3355.

- McCoy, A. N., & Platt, M. L. (2005). Risk-sensitive neurons in macaque posterior cingulate cortex. *Nature Neuroscience*, 8, 1220–1227.
- Samuelson, P. (1963). Risk and uncertainty: a fallacy of large numbers. *Scientia*, 98, 108–113.
- Sarin, R. K., & Weber, M. (1993). Risk–value models. *European Journal of Operations Research*, 70, 135–149.
- Shafir, S., Bechar, A., & Weber, E. U. (2003). Cognition-mediated coevolution: context-dependent evaluations and sensitivity of pollinators to variability in nectar rewards. *Plant Systematics and Evolution*, 238, 195–209.
- Shafir, S., Reich, T., Tsur, E., Erev, I., & Lotem, A. (2008). Perceptual accuracy and conflicting effects of certainty on risk-taking behaviour. *Nature*, 453, 917–920.
- Starmer, C. (2000). Developments in non-expected utility theory: the hunt for a descriptive theory of choice under risk. *Journal of Economic Literature*, 38, 332–382.
- Tversky, A., & Kahneman, D. (1992). Advances in prospect theory: cumulative representation of uncertainty. *Journal of Risk and Uncertainty*, 5, 297–323.
- von Neumann, J., & Morgenstern, O. (1947). *Theory of games and economic behavior* (2nd ed.). Princeton, NJ: Princeton University Press.
- Weber, B., Rangel, A., Wibral, M., & Falk, A. (2009). The medial prefrontal cortex exhibits money illusion. *Proceedings of the National Academy of Science*, 106, 5025–5028.
- Weber, E. U. (2004). Perception matters: psychophysics for economists. In J. Carrillo, & I. Brocas (Eds.), *Psychology and economics* (pp. 165–176). Oxford, UK: Oxford University Press.
- Weber, E. U., & Johnson, E. J. (2009). Mindful judgment and decision making. *Annual Review of Psychology*, 60, 53–86.
- Weber, E. U., & Hsee, C. K. (1998). Cross-cultural differences in risk perception but cross-cultural similarities in attitudes towards risk. *Management Science*, 44, 1205–1217.
- Weber, E. U., & Milliman, R. (1997). Perceived risk attitudes: relating risk perception to risky choice. *Management Science*, 43, 122–143.
- Weber, E. U., Shafir, S., & Blais, A. (2004). Predicting risk sensitivity in humans and lower animals: risk as variance or coefficient of variation. *Psychological Review*, 111, 430–445.
- Zipf, G. K. (1949). *Human behavior and the principle of least effort*. New York: Addison-Wesley Press.